

# Post-MAP migration of crosswell seismic data

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## SUMMARY

We present a method that approximately collapses diffraction events in crosswell seismic data that have been mapped using a point-to-point VSP-CDP algorithm and stacked. Our approach uses a conventional surface seismic migration algorithm with a modified velocity to compensate for the effects of VSP-CDP mapping and the crosswell geometry. The technique is approximate, and is most effective near the midpoint between the wells. As the reflection point moves toward either well, the approximations break down and the diffractions are not well collapsed. However, the diffractions are more focused with approximate migration than without. For continuous horizontal reflections, the VSP-CDP mapped section and the approximate post-MAP migration section are almost identical.

## INTRODUCTION

There are two schools of thought in imaging crosswell reflection data. One approach (e.g. Goult, 1991, Schuster, 1993) is to use prestack migration optimally collapse diffraction energy. Another approach (Lazaratos, et al, 1993) is to use a point-to-point VSP-CDP mapping algorithm (Wyatt and Wyatt, 1981). Although the VSP-CDP algorithm does not correctly handle diffraction events, it is a more benign operation in that it does not smear noise along an imaging ellipse. Since prestack crosswell reflection data are often of low signal-to-noise ratio, due to both coherent noise (tube waves, conversions, guided waves, etc.) and incoherent noise, prestack migration may not be appropriate. Prestack migration is only used in surface seismic data processing when the signal-to-noise ratio is high.

In this paper, we present a method to approximately collapse the diffractions present in the VSP-CDP mapped and stacked crosswell reflection image. This approach is similar to post-stack migration of surface seismic data. However, unlike surface reflection data, where we transform the image from unmigrated time to migrated time (or depth), we transform the crosswell data from unmigrated, but mapped depth, to (approximately) migrated depth.

## THEORY

For two vertical wells, the traveltimes equations for a diffractor at  $(x_d, z_d)$  in a common middepth gather (CMG) is

$$t_{diff} = \frac{1}{v_{diff}} \left\{ \sqrt{x_d^2 + [(Z_d - M) + O]^2} + \sqrt{(x_w - x_d)^2 + [(Z_d - M) - O]^2} \right\}$$

where,  $v_{diff}$  is the mean ray-path velocity,

$x_w$  is the distance between two wells,

$M$  is the middepth  $\left( \frac{Z_R + Z_S}{2} \right)$ ,

$O$  is the offset  $\left( \frac{Z_R - Z_S}{2} \right)$ ,

$Z_S$  is the source depth,

and  $Z_R$  is the receiver depth.

If the diffractor is located midway between the two wells, the traveltimes equation is

$$t_{diff} v_{diff} = \sqrt{\left( \frac{x_w}{2} \right)^2 + [(Z_d - M) + O]^2} + \sqrt{\left( \frac{x_w}{2} \right)^2 + [(Z_d - M) - O]^2} \quad (1)$$

For small  $O$ ,  $t_{diff}^2 v_{diff}^2$  can be approximated as

$$t_{diff}^2 v_{diff}^2 \approx x_w^2 + 4(Z_d - M)^2 + 2O^2, \quad (2)$$

and the squared minimum traveltimes,  $t_{diff,0}^2$ , is

$$t_{diff,0}^2 = \frac{x_w^2 + 4(Z_d - M)^2}{v_{diff}^2}. \quad (3)$$

Thus,

$$t_{diff}^2 = t_{diff,0}^2 + \frac{2O^2}{v_{diff}^2}. \quad (4)$$

As shown by Eq. (4), the traveltimes curve for a diffractor in a CMG can be approximated by a hyperbola. The caveats for this approximation are that the offsets are small and the diffraction is near the midpoint between the wells. For typical crosswell reflection incidence angles (30~70°), these caveats may be replaced by one – namely that the diffraction point is near the midpoint.

VSP-CDP transforms common middepth gathers into a mapped section  $(x_m, z_m)$  by the equations (Lazaratos, 1993):

$$x_m = \frac{x_w}{2} \left[ 1 + \left( \frac{2O}{\sqrt{v^2 t^2 - x_w^2}} \right) \right] \quad \text{and} \quad (5)$$

$$z_m = \frac{1}{2} \left( \sqrt{v^2 t^2 - x_w^2} + 2M \right). \quad (6)$$

For a diffractor at the midpoint between wells, the equation of the diffractor after VSP-CDP mapping can be obtained by combining Eq. (2), (5), and (6). If only upgoing waves are considered, then

$$z_m = M + \frac{Z_d - M}{\sqrt{1 - \left( \frac{\sqrt{2} x_m}{x_w} - \frac{1}{\sqrt{2}} \right)^2}}. \quad (7)$$

The minimum value of  $z_m$  is the depth of the diffractor,  $Z_d$ .

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Let  $T$  be the distance from the midpoint normalized by the half distance between wells:

$$T = \frac{x_m - \frac{x_w}{2}}{\frac{x_w}{2}} \quad (-1 < T < 1) \quad (8)$$

Now

$$z_m = M + \frac{Z_d - M}{\sqrt{1 - \frac{T^2}{2}}} \quad (9)$$

where

$$z_m = Z_d \quad \text{at } T = 0 \left( x_m = \frac{x_w}{2} \right) \quad \text{and}$$

$$z_m = M + \sqrt{2}(Z_d - M) \quad \text{at } T = \pm 1 \left( x_w = 0 \text{ or } x_w \right).$$

For small  $T$ ,  $z_m$  can be approximated by

$$z_m \approx Z_d + (Z_d - M) \cdot \frac{T^2}{4} \quad (10)$$

If  $z_m$  is normalized by  $Z_d$ , then

$$\frac{z_m}{Z_d} = \frac{M}{Z_d} + \frac{1 - M/Z_d}{\sqrt{1 - \frac{T^2}{2}}} \quad \text{and} \quad (11)$$

$$\frac{z_m}{Z_d} \approx 1 + \left(1 - \frac{M}{Z_d}\right) \cdot \frac{T^2}{4} \quad (12)$$

If we call the variation of  $z_m$  with  $T$   $z_{NMO}$ , then

$$z_{NMO} = z_m - Z_d = (Z_d - M) \cdot \frac{T^2}{4} \quad \text{and} \quad (13)$$

$$\frac{z_{NMO}}{Z_d} = \left(1 - \frac{M}{Z_d}\right) \cdot \frac{T^2}{4} \quad (14)$$

As shown Eq. (13) and Eq. (14), the moveout equation for a diffractor located at the center of wells in the mapped section can be described as a parabola with apex at  $\left(\frac{x_w}{2}, Z_d\right)$  for small  $T$ . Also, the moveout decreases as  $M$  is close to  $Z_d$  for specific  $Z_d$ . The moveout as a function of  $M$  is smaller for small  $T$  and larger near the wells. Thus, the stacking process doesn't significantly affect the frequency content of the diffractions near the midpoint but stacking will tend to lowpass filter diffraction energy mapped near the wells. Fig. 1 shows moveouts in the mapped section computed using Eq. (11) and (12) where  $M$  is  $0.77 Z_d$ . As shown the figure, Eq. (12) approximates well Eq. (11) for  $x_m$  within the distance  $\frac{x_w}{4}$  from the midpoint ( $|T| < 0.5$ ).

Thus conventional post-stack migration for surface seismic data can be used to collapse the parabola from a diffractor (midway between the two wells) in the VSP-CDP mapped section by modifying the migration velocity. If Eq. (13) is converted to two-way traveltine, then

$$T_{NMO} = (Z_d - M) \cdot \frac{T^2}{4} \cdot \frac{2}{v_{diff}} \quad (15)$$

In a surface seismic geometry, the moveout for a diffractor at  $(T=0, Z_d)$  in a zero-offset section is

$$T_{NMO} = \frac{T^2 x_w^2}{4 Z_d v^*} \quad (16)$$

The velocity for post-stack migration used to collapse the curve from a diffractor in the VSP-CDP mapped section can be obtained by making Eq. (16) equal to Eq. (15):

$$v^* = \frac{x_w^2}{2 Z_d (Z_d - M)} \cdot v_{diff} \quad (17)$$

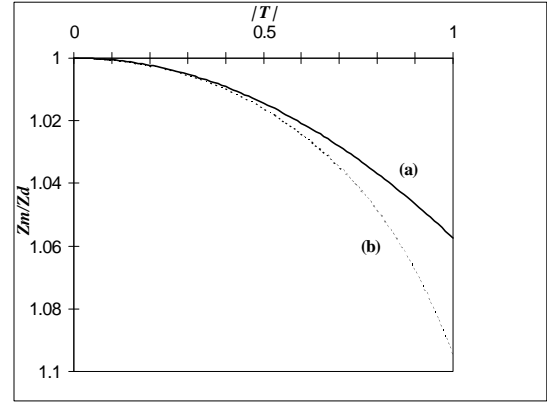


Fig. 1. Moveouts in the mapped section are derived  
(a) by the approximate parabola equation (12)  
(b) by the traveltine equation (11)  
where  $M$  is  $0.77 Z_d$ .

## EXAMPLES

The above theory was applied to two velocity models. Synthetic seismograms for both models were made using a finite difference modeling method with the acoustic wave equation. The first model consisted of two reflectors and three diffractors between them (Fig. 2(b)). Each diffractor was located at a different position and a different depth. One was 100 ft from the source well, another was midway between the two wells, and the third was 100 ft from the receiver well. The distance between source well and receiver well was 400 ft. Sources and receivers were located between 1900 and 2900 ft. Both source and receiver intervals were 25 ft. The source was a zero-phase Ricker wave with a center frequency 300 Hz. Fig. 2(a) is the result of applying the VSP-CDP mapping, stacking middepths from 1912.5 to 2150 ft. As shown in the figure, the three diffractions were not collapsed by VSP-CDP mapping. The mapped diffractions can be misinterpreted as dipping or discontinuous horizontal reflectors. Fig. 2(c) shows the post-stack migrated section corresponding to the VSP-CDP mapped section in Fig. 2(a). After migration, dip filtering

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was used to reduce edge effects from the short aperture of the parabola curve. The velocity for migration was calculated with Eq. (17) using a middepth of 2150 ft. As shown in the figure, the curve for a diffractor at the center was collapsed perfectly. Curves for the other diffractors are collapsed even though there is some mispositioning and residual tails. These effects come from the assumption of a midpoint diffractor.

The second model consisted of many horizontal reflectors. This model was used to verify the fact that post-stack migration doesn't distort the VSP-CDP mapping for horizontal reflectors. The distance between the source well and receiver well was 586 ft. Sources and receivers were located from 2400 to 3000 ft and both source and receiver intervals were 5 ft (Fig. 3(b)). The source was the zero phase Ricker wave with the center frequency 700 Hz. Fig. 3(a) shows the VSP-CDP mapped and stacked section for this velocity model. The mapped data were stacked using middepths from 2450 to 2900 ft. The post-MAP migrated section is shown in Fig. 3(d).  $M$  in Eq. (17) was assumed  $0.83 Z_d$  and the velocities used in the VSP-CDP mapping were used as  $v_{diff}$ . The image is a little bit distorted near the wells. Fig. 3(c) was created from three separate parts. The migrated section (Fig. 3(d)) was used *only* for the center part (100~486 ft from the source well). The VSP-CDP mapped section was used for the region near the wells (100 away from each well) where the migration assumptions break down. In this region, the Fresnel zone is smaller (Lazaratos, 1993) and migration is less necessary. These boundaries are indicated as dotted lines in the figure. As shown in the figure, the application of the post-stack migration to the VSP-CDP mapped section doesn't distort the result for

horizontal reflectors.

### CONCLUSIONS

Crosswell diffractions are not collapsed by the VSP-CDP mapping. These diffractions can be misinterpreted as horizontal or dipping reflectors. We focused diffractions using conventional surface seismic post-stack migration with a modified velocity. In two model datasets, the results are superior to VSP-CDP stacking.

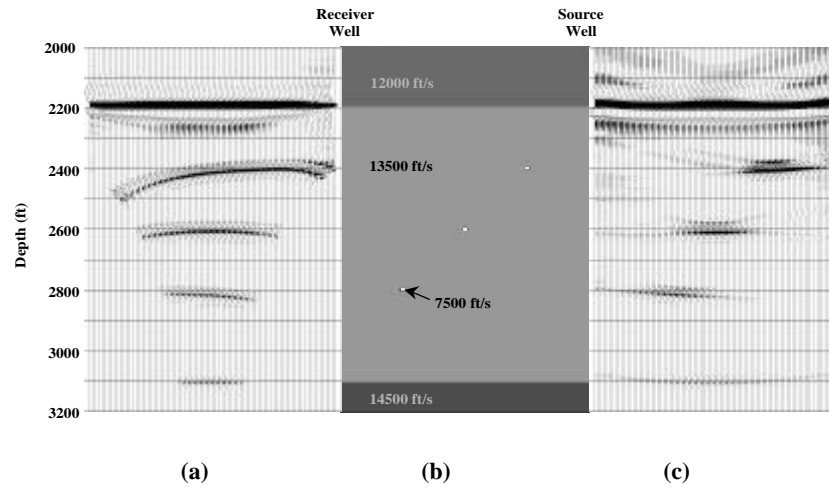
### ACKNOWLEDGEMENTS

The authors would like to thank to the Department of Energy (Contract # 00472-94-I) and the Gas Research Institute (Contract # 5094-210-3181) for support of this research.

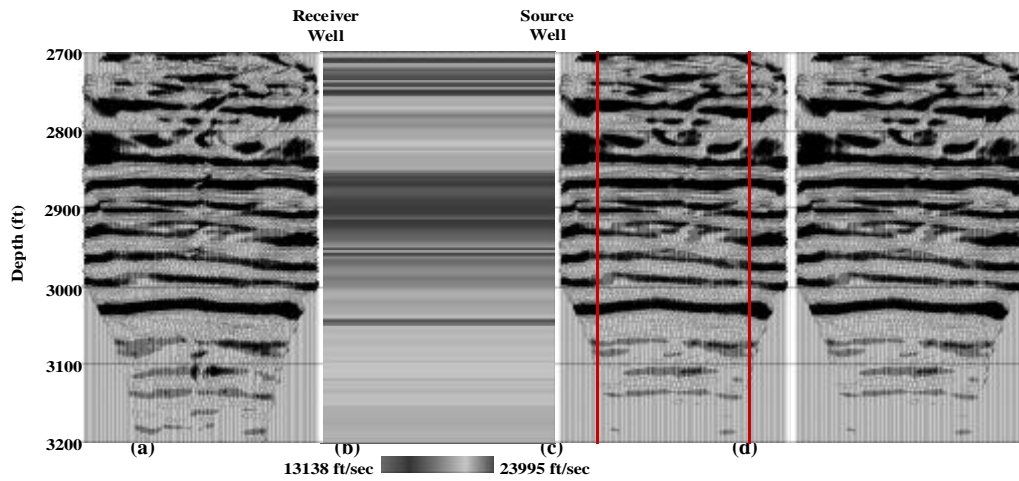
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**Fig. 2. (a)** VSP-CDP mapped section, stacking middepth from 1912.5 to 2150 ft after applying the VSP-CDP mapping to common middepth gathers.  
**(b)** Velocity model of three diffractors and two horizontal reflectors.  
**(c)** Post-stack migrated section of the section (a).



**Figure 3. (a)** VSP-CDP mapped section of the section stacked with common middepth gathers (Middepth: 2450 ~ 2900 ft).  
**(b)** Velocity model of horizontal reflectors.  
**(c)** Combination of VSP-CDP mapped section (a) and post-stack migration section (d).  
**(d)** Post-stack migrated section of the section (a).